Explanation of Plate I.

Fig. 1. A piece of the placental cord of *Zygæna blochii*, natural size.

Fig. 2. Transverse section through the same, showing artery and vein, lymphatic (?) spaces, and three appendicula in oblique section with parts of two more in vertical section. × 16.

Fig. 3. A portion of one of the appendicula of the same, showing the ramifying vessel. × 21.

Fig. 4. Transverse section through part of one of the appendicula of the same, near its base. × 110.

Fig. 5. Transverse section through uterine wall of *Myliobatis nieuhofii*, showing fibrous and muscular coats, and mucous membrane, with the bases of three papillæ. × 21.

Fig. 6. Obliquely transverse section through part of one of the uterine papillæ of the same, showing some of the simple follicles of the mucous membrane in oblique section, and one of the racemose follicles. × 110.

III.—*On Clebsch's Transformation of the Hydrokinetic Equations.* *By* Asutosh Mukhopadhyay, M. A., F. R. A. S., F. R. S. E.

[Received February 27th;—Read March 6th, 1889.]

A first integral of the hydrokinetic equations of Euler may be obtained by known methods in three cases: (1) Irrotational motion; (2) Steady rotational motion; (3) General rotational motion. It is the object of this note to show how the method of applying Clebsch's transformation to the third case can be materially simplified, and incidentally the relation between the three solutions is pointed out.*

Starting, then, with the hydrokinetic equations, we remark that they may be at once reduced to the forms

$$\frac{du}{dt} - 2v\zeta + 2w\eta + \frac{dR}{dx} = 0 \qquad \ldots\ldots\ldots\ldots \quad (1)$$

$$\frac{dv}{dt} - 2w\xi + 2u\zeta + \frac{dR}{dy} = 0 \qquad \ldots\ldots\ldots\ldots \quad (2)$$

$$\frac{dw}{dt} - 2u\eta + 2v\xi + \frac{dR}{dz} = 0 \qquad \ldots\ldots\ldots\ldots \quad (3)$$

where

$$R = \int \frac{dp}{\rho} + V + \frac{1}{2} q^2$$

$$q^2 = u^2 + v^2 + w^2$$

* For the ordinary method, see Basset's Hydrodynamics, vol. i, p. 28.

In the first case, for irrotational motion, the components of molecular rotation ξ, η, ζ vanish, implying the equations

$$u = \frac{d\phi}{dx}, \quad v = \frac{d\phi}{dy}, \quad w = \frac{d\phi}{dz}$$

and the equations of motion reduce to

$$\frac{dU}{dx} = 0, \quad \frac{dU}{dy} = 0, \quad \frac{dU}{dz} = 0$$

where

$$U = \frac{d\phi}{dt} + R.$$

Hence, the required first integral is

$$\int \frac{dp}{\rho} + V + \frac{1}{2}q^2 + \frac{d\phi}{dt} = F,$$

where F is ordinarily a function of the time, but for steady motion an absolute constant throughout the liquid.

Secondly, if the motion is rotational but steady, we have

$$\frac{du}{dt} = 0, \quad \frac{dv}{dt} = 0, \quad \frac{dw}{dt} = 0$$

and the equations of motion lead to

$$u\frac{dR}{dx} + v\frac{dR}{dy} + w\frac{dR}{dz} = 0$$

$$\xi\frac{dR}{dx} + \eta\frac{dR}{dy} + \zeta\frac{dR}{dz} = 0.$$

These linear differential equations lead, by Laplaces's method, to the subsidiary systems

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

$$\frac{dx}{\xi} = \frac{dy}{\eta} = \frac{dz}{\zeta}$$

which denote respectively stream lines and vortex lines. Hence, it is possible to construct a series of surfaces

$$R = \text{constant}$$

each of which shall be covered over with a net work of stream lines and vortex lines. Hence for steady rotational motion we have

$$\int \frac{dp}{\rho} + V + \frac{1}{2}q^2 = \text{constant},$$

the constant being an absolute constant so long as we pass from point to point on a stream line or vortex line, but which varies as we pass from one stream line to another or from one vortex line to another.

Thirdly, if the motion of the liquid is perfectly general, neither steady nor irrotational, we may put, after Clebsch,

$$udx + vdy + wdz = d\phi + \lambda\, d\chi.$$

Observe for a moment that as this simply signifies that the differential expression on the lefthand side, when not a perfect differential may be resolved into two, one of which is so, and the other may be made so by means of an integrating factor, the legitimacy of the transformation is selfevident. We have then

$$u = \frac{d\phi}{dx} + \lambda \frac{d\chi}{dx},\ v = \frac{d\phi}{dy} + \lambda \frac{d\chi}{dy},$$

$$w = \frac{d\phi}{dz} + \lambda \frac{d\chi}{dz},$$

furnishing the known expressions

$$2\,\xi = \frac{d\lambda}{dy}\frac{d\chi}{dz} - \frac{d\lambda}{dz}\frac{d\chi}{dy}$$

$$2\,\eta = \frac{d\lambda}{dz}\frac{d\chi}{dx} - \frac{d\lambda}{dx}\frac{d\chi}{dz}$$

$$2\,\zeta = \frac{d\lambda}{dx}\frac{d\chi}{dy} - \frac{d\lambda}{dy}\frac{d\chi}{dx}$$

These lead to the equations

$$\xi \frac{d\lambda}{dx} + \eta \frac{d\lambda}{dy} + \zeta \frac{d\lambda}{dz} = 0$$

$$\xi \frac{d\chi}{dx} + \eta \frac{d\chi}{dy} + \zeta \frac{d\chi}{dz} = 0$$

both of which give the subsidiary system

$$\frac{dx}{\xi} = \frac{dy}{\eta} = \frac{dz}{\zeta}$$

the differential equation of vortex lines. Hence the vortex lines are obtained as the intersection of the surfaces λ = constant, χ = constant. Again, the value of u gives

$$\frac{du}{dt} = \frac{d}{dx}\left(\frac{d\phi}{dt} + \lambda \frac{d\chi}{dt}\right) + \frac{d\lambda}{dt}\frac{d\chi}{dx} - \frac{d\lambda}{dx}\frac{d\chi}{dt}.$$

Substituting in equation (1), we have *at once*

$$\frac{dH}{dx} + \frac{\delta\lambda}{\delta t}\frac{d\chi}{dx} - \frac{\delta\chi}{\delta t}\frac{d\lambda}{dx} = 0$$

where

$$H = \int \frac{dp}{\rho} + V + \frac{d\phi}{dt} + \lambda \frac{d\chi}{dt} + \frac{1}{2} q^2,$$

and δ denotes particle differentiation. Equations (2) and (3) lead to two similar equations, and we have

$$\xi \frac{dH}{dx} + \eta \frac{dH}{dy} + \zeta \frac{dH}{dz} = 0$$

leading to the subsidiary system

$$\frac{dx}{\xi} = \frac{dy}{\eta} = \frac{dz}{\zeta}$$

which denote vortex lines. Hence, we see that it is possible to construct a family of surfaces

$$H = \text{constant},$$

covered over by vortex lines, and the mode of integration shows *immediately* that the constant is a function of the time alone. Therefore, for steady rotational motion we have

$$\int \frac{dp}{\rho} + V + \frac{d\phi}{dt} + \lambda \frac{d\chi}{dt} + \frac{1}{2} q^2 = F(t)$$

along a vortex line.

IV.—*Note on Stokes's Theorem and Hydrokinetic Circulation.*

By Asutosh Mukhopadhyay, M. A., F. R. A. S., F. R. S. E.

[Received March 24th;—Read April 3rd, 1889.]

The object of this note is to give a new proof of Stokes's formula for hydrokinetic circulation

$$\int (udx + vdy + wdz) = 2 \int\int (l\,\xi + m\,\eta + n\,\zeta)\, d\,S,$$

and to point out how it is an immediate consequence of the theory of the change of the variables in a multiple integral.

Assume, after Clebsch,

$$udx + vdy + wdz = d\phi + \lambda\, d\chi,$$

so that the integration being performed round a closed curve, we have

$$\int (udx + vdy + wdz) = \int \lambda\, d\chi.$$

But, the value of

$$\int \lambda\, d\chi$$

taken round the closed curve is clearly equal to the sum of the values of

$$\int\int d\lambda\, d\chi$$

taken round the projections of the closed curve on the coordinate planes. Now, for the projected curve on the coordinate plane of yz, we have at once from the ordinary formulæ for the transformation of multiple integrals,

$$\int\int d\lambda\, d\chi$$

$$= \int\int \left(\frac{d\lambda}{dy}\frac{d\chi}{dz} - \frac{d\lambda}{dz}\frac{d\chi}{dy}\right) dy\, dz.$$

The projected curves on the other two coordinate planes lead to two similar expressions. Hence, the circulation round the given closed curve is furnished by

$$\int (udx + vdy + wdz)$$

$$= \int\int \left(\frac{d\lambda}{dy}\frac{d\chi}{dz} - \frac{d\lambda}{dz}\frac{d\chi}{dy}\right) dy\, dz$$

$$+ \int\int \left(\frac{d\lambda}{dz}\frac{d\chi}{dx} - \frac{d\lambda}{dx}\frac{d\chi}{dz}\right) dz\, dx$$

$$+ \int\int \left(\frac{d\lambda}{dx}\frac{d\chi}{dy} - \frac{d\lambda}{dy}\frac{d\chi}{dx}\right) dx\, dy.$$

But, as an immediate consequence of Clebsch's transformation, we have

$$u = \frac{d\phi}{dx} + \lambda\frac{d\chi}{dx}$$

$$v = \frac{d\phi}{dy} + \lambda\frac{d\chi}{dy}$$

$$w = \frac{d\phi}{dz} + \lambda\frac{d\chi}{dz},$$

whence

$$2\xi = \frac{dw}{dy} - \frac{dv}{dz} = \frac{d\lambda}{dy}\frac{d\chi}{dz} - \frac{d\lambda}{dz}\frac{d\chi}{dy}$$

$$2\eta = \frac{du}{dz} - \frac{dw}{dx} = \frac{d\lambda}{dz}\frac{d\chi}{dx} - \frac{d\lambda}{dx}\frac{d\chi}{dz}$$

$$2\zeta = \frac{dv}{dx} - \frac{du}{dy} = \frac{d\lambda}{dx}\frac{d\chi}{dy} - \frac{d\lambda}{dy}\frac{d\chi}{dx}.$$

Therefore, putting

$$dy\,dz = l\,dS, \quad dx\,dz = m\,dS, \quad dx\,dy = n\,dS,$$

where l, m, n are the direction cosines of the normal, we have

$$\int (udx + vdz + wdz)$$

$$= \int\int \left\{ l\left(\frac{dw}{dy} - \frac{dv}{dz}\right) + m\left(\frac{du}{dz} - \frac{dw}{dx}\right) + n\left(\frac{dv}{dx} - \frac{du}{dy}\right) \right\} dS$$

$$= 2 \int\int (l\xi + m\eta + n\zeta)\, dS,$$

which is Stokes's Theorem. It is worth noting that as no physical conception enters into the above proof, it holds good whether we regard the theorem as a purely analytical one or as merely furnishing a formula for hydrokinetic circulation.

V.—*On a Curve of Aberrancy.*

By Asutosh Mukhopadhyay, M. A., F. R. A. S., F. R. S. E.

[Received May 23rd ;—Read June 5th, 1889.]

If a curve be referred to rectangular axes drawn through any origin, the coordinates (α, β) of the centre of aberrancy, which is the centre of the osculating conic at any given point (x, y) of the curve, are given in the most general form by the system

$$\alpha = x - \frac{3qr}{3qs - 5r^2}$$

$$\beta = y - \frac{3q\,(pr - 3q^2)}{3qs - 5r^2}$$

where p, q, r, s are the successive differential coefficients of y with respect to x.* The locus of (α, β) is called the aberrancy curve of the given curve, and in this note, I shall investigate the aberrancy curve of a plane cubic of Newton's fourth class†

$$y = ax^3 + 3bx^2 + 3cx + d$$

in which the diametral conic degenerates into the line at infinity.

We have

$$p = 3\,(ax^2 + 2bx + c)$$
$$q = 6\,(ax + b)$$
$$r = 6a$$
$$s = 0$$

* J. A. S. B. 1888, vol. lvii, part ii, p. 324.

† Salmon's Higher Plane Curves, (Ed. 1879), p. 177.

whence

$$pr - 3q^2 = 18\,(ac - b^2) - 90\,(ax + b)^2$$

$$\alpha = \frac{8x}{5} + \frac{3b}{5a}$$

$$\beta = y + \frac{ax + b}{10a^2}\left\{ 18\,(ac - b^2) - 90\,(ax + b)^2 \right\}$$

Therefore

$$x = \frac{3\alpha}{8} - \frac{3b}{8a}$$

$$ax + b = \frac{5}{8}\,(a\alpha + b)$$

and

$$y = \beta - \frac{9\,(a\,\alpha + b)}{8a^2}\left\{ (ac - b^2) - \frac{125}{64}\,(a\alpha + b)^2 \right\}$$

But from the equation of the curve we have

$$a^2 y = (ax + b)^3 + 3a\,(ac - b^2)\,x + a^2 d - b^3.$$

Therefore, substituting for x and y in terms of α and β, we have

$$64\,a^2\beta = -\,125\,a^3\alpha^3 - 375\,a^2 b\alpha^2 + (192\,ac - 567b^2)\,a\alpha + (64a^2 d - 189b^3),$$

or, writing x, y for α, β, we see that the aberrancy curve of the plane cubic

$$y = ax^3 + 3bx^2 + 3cx + d$$

is another plane cubic of the same class

$$y = Ax^3 + 3Bx^2 + 3Cx + D$$

where

$$A = -\,ka$$

$$B = -\,kb$$

$$C = -\,kc + (1 + k)\,\frac{ac - b^2}{a}$$

$$D = -\,kd + (1 + k)\,\frac{a^2 d - b^3}{a^2}$$

$$k = \frac{125}{64}.$$

If, therefore,

$$H = ac - b^2,\; G = a^2 d - 3abc + 2b^3$$

be the invariants of the given cubic, and H', G' the corresponding quantities for the aberrancy cubic, *viz.*,

$$H' = AC - B^2,\; G' = A^2 D - 3ABC + 2B^3,$$

we have by direct calculation

$$H' = -\,kH$$

$$G' = k^2 G.$$

It follows, therefore, that the quantity

$$\frac{H^2}{G} = \frac{(ac - b^2)^2}{a^2d - 3abc + 2b^3}$$

is an invariant for the given cubic and its aberrancy curve.

If we seek the common points of intersection of the two cubics, we find on subtracting the equations

$$(ax + b)^3 = 0$$

which shews that the two cubics have only one common point of intersection which is the point of inflexion for both; the coordinates of the point are

$$x = -\frac{b}{a}, \quad y = \frac{G}{a^2}$$

VI.—*Natural History Notes from H. M. Indian Marine Survey Steamer* 'Investigator,' *Commander* Alfred Carpenter, R. N., D. S. O., *commanding.*—No. 15. *Descriptions of seven additional new Indian Amphipods.*—*By* G. M. Giles, M. B., F. R. C. S., *late Surgeon-Naturalist to the Survey.*

[Received and Read November 6th, 1889.]

(With Plate II.)

Before proceding to the description of the species now described, I have to make a correction in my last paper read on February 1st, 1888.

In that communication, I described, under the name of *Concholestes dentallii*, gen. et sp. nov., a curious corophiid which inhabits deserted dentalium shells; remarking that I believed that such a habit had not beeu previously noted in an amphipod. I find, however, I was in error in this matter, as, while searching for references to species which might be identical with those described in the present paper, I came across a description of a Norwegian species which is certainly congeneric and, like the Indian species, inhabits deserted dentalium shells. Sars (Forh. Vidensk.-Selsk. Christiania, 1882, No. 18, pp. 113, Part VI, fig. 7) describes this species as *Siphonœcetes pallidus.*

I do not see, however, how either Sars' or my species can be included in *Siphonœcetes* without unduly straining Kroyer's definition of the genus in Nat. Tidskr. I, p. 491. In the two species under consideration, the 1st and 2nd gnathopoda, instead of being subequal, present a very marked difference of size; and again, the eighth thoracic appendages are very long, instead of the 6th, 7th, and 8th being "very short." My species too wants the double hook to the single ramus of the last